The Optimum Tilt Angles and Orientations of PV Claddings for Building-Integrated Photovoltaic (BIPV) Applications

Hongxing Yang
Associate Professor
Department of Building Services Engineering,
The Hong Kong Polytechnic University,
Kowloon, Hong Kong
e-mail: behxyang@polyu.edu.hk

Lin Lu
Assistant Professor
Department of Building Services Engineering,
The Hong Kong Polytechnic University,
Hong Kong
e-mail: bellu@polyu.edu.hk

The tilt and azimuth angles of a photovoltaic (PV) array affect the amount of incident solar radiation exposed on the array. This paper develops a new mathematical model for calculating the optimum tilt angles and azimuth angles for building-integrated photovoltaic (BIPV) applications in Hong Kong on yearly, seasonal, and monthly bases. The influence of PV cladding orientation on the power output of PV modules is also investigated. The correlations between the optimum tilt angle and local weather conditions or local environmental conditions are investigated. The results give reasonable solutions for the optimum tilt angles for BIPV applications for both grid-connected and stand-alone systems. [DOI: 10.1115/1.2212439]

1 Introduction

The performance of photovoltaic (PV) modules and building-integrated photovoltaic (BIPV) systems is highly influenced by the modules' orientations. The PV modules must be oriented and tilted to gain the maximum solar radiation and to avoid unwanted shading. Most former studies treat the problem qualitatively and quantitatively for a certain location. For solar energy applications, the optimum orientation is usually suggested to be south-facing in the northern hemisphere, and the optimum tilt angle depends only on the local latitude, $\beta_{\text{opt}}=f(\phi)$. For example, Duffie and Beckman [1] suggested that $\beta_{\text{opt}}=(\phi+15\text{ deg})\pm 15$ deg and Lewis [2] suggested $\beta_{\text{opt}}=\phi\pm 8$ deg (where ϕ is the latitude of the location). For a specific area, Asl-Soleimani [3] reported that the optimum tilt angle is 30 deg in Tehran, lower than the local latitude of $\phi=35.7$ deg for a grid-connected PV system to obtain maximum yearly energy generation. Christensen and Barker [4] found that surface tilt angles and azimuth angles can be varied over a considerable range without significantly reducing the amount of annual incident solar radiation.

There are limitations of the former quantitative studies. The hourly clearness index was not taken into account, and simplified sky models were usually used, while precise anisotropic sky models can give us more accurate results as stated by Lu [5]. Besides, for the same latitude area, clearness index and its yearly distribution can be quite different, so that the optimum tilt angles can be totally different. This paper aims to develop a new practical method (considering hourly clearness index) to find the relationship between orientation and absorbed solar irradiance and to obtain the local installation orientation priority and optimum tilt angles of PV modules in terms of yearly, seasonal, and monthly demands.

Contributed by Solar Energy Division of ASME for publication in the Journal of Solar Energy Engineering. Manuscript received August 26, 2004; final manuscript received November 14, 2005. Review conducted by Andy Walker.

2 Mathematical Method for the Optimum Slope

The total hourly solar irradiance (i) on a tilted plane can be described as

$$G_{tt}(i)=G_{bt}(i)+G_{dt}(i)+G_r(i) \tag{1}$$

where $G_{tt}(i)$ is the total solar irradiance on a tilted plane at hour i, $\text{W/m}^2\,\text{h}$; G_{bt} is the total beam solar irradiance absorbed by the inclined plane, $\text{W/m}^2\,\text{h}$; G_{dt} is the total diffuse solar irradiance absorbed by the inclined plane, $\text{W/m}^2\,\text{h}$; and G_r is the total reflected solar irradiance absorbed by the inclined plane.

For a fixed orientation, the optimum tilt angle comes by solving the following equation for β

$$\frac{d}{d\beta}\left(\sum_{i=1}^{m} G_{tt}(i)\right)_{\beta_{\text{opt}}}=0 \tag{2}$$

where m is the number of hours when the calculation period ends.

The beam part G_{bt} can be simulated by

$$G_{bt}=G_{bh}\frac{\cos\theta}{\cos\theta_z}=G_{bh}R_b \tag{3}$$

where G_{bh} is the beam irradiance on a horizontal plane; θ is the angle of incidence, the angle between the beam radiation on a surface and the normal to that surface; θ_z is the angle of incidence for horizontal surfaces, which is called the zenith angle of the sun as well; and R_b is the geometric factor. For the calculation of the angle of incidence, the equations from Duffie and Beckman [1] are used.

The reflected part G_r can be simulated by the following formula

$$G_r=\frac{\rho_0}{2}G_{th}(1-\cos\beta) \tag{4}$$

where G_{th} is the total irradiance on a horizontal plane, and ρ_0 is the ground reflectance. If there is snow on the ground, the surface albedo is set to be 0.6 (albedo for snow ranges from about 0.35 for old snow to 0.95 for dry new snow). If no snow is indicated, the surface albedo is set to be 0.2, a nominal value for green vegetation and some soil types.

The Reindl [6] model for titled surface diffuse radiation is used for diffuse radiation part

$$G_{dt}=0.5G_{dh}[1+\cos(\beta)](1-A_I)\left(1+f\times\sin^3\left(\frac{\beta}{2}\right)\right)+G_{dh}A_IR_b \tag{5}$$

where G_{dh} is the diffuse irradiance on a horizontal plane; $A_I=G_{bn}/G_{on}=(G_{bh}/\cos\theta_z)/(G_0/\cos\theta_z)=G_{bh}/G_0$; and

$$f=\sqrt{\frac{G_{bh}}{G_{th}}} \tag{6}$$

$$G_0=G_{sc}\left[1+0.033\cos\left(\frac{360n}{365}\right)\right](\cos\delta\cos\phi\cos\omega+\sin\delta\sin\phi) \tag{7}$$

where G_{sc} is the solar constant, 1353 W/m^2.

For the above models, G_{bh} and G_{dh} are necessitated, while most observatories only offer hourly total solar radiation on a horizontal plane. Thus, suitable models for dividing the total hourly radiation into beam and diffuse components are required. According to the three-section and piecewise linear function relating the hourly dif-

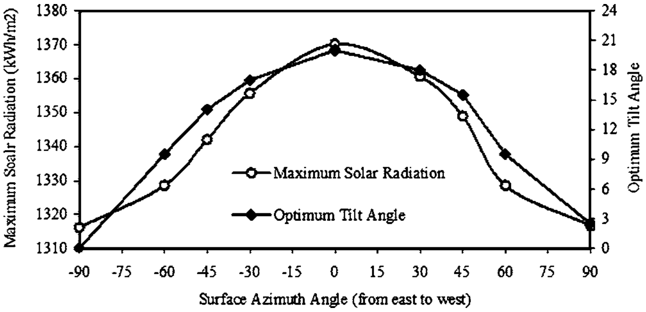

Fig. 1 Optimum slopes and maximum yearly solar radiation profiles

fuse ratio G_{bh}/G_{th} to the clearness index k_T, established by Orgill and Hollands [7], Yik et al. [8] developed the correlation between hourly diffuse ratio and clearness index for Hong Kong, which will be used in the local study. The weather data of the year 1989 are used for solar energy system simulation as discussed by Yang [9].

3 Results and Discussions

3.1 Yearly Optimum Tilt Angle. Finding the local yearly optimum tilt angles for various azimuth angles is important in determining the optimum orientation. In this study, typical popular azimuth angles for the northern hemisphere are east-facing ($\gamma = -90$ deg), south-east facings ($\gamma = -60$ deg, $\gamma = -45$ deg, and $\gamma = -30$ deg), south-facing ($\gamma = 0$ deg), south-west facings ($\gamma = 30$ deg, $\gamma = 45$ deg, and $\gamma = 60$ deg), and west-facing ($\gamma = 90$ deg). Figure 1 shows the yearly optimum tilt angles and the absorbed solar radiation for different orientations.

The maximum point comes at south-facing azimuth angle with 20 deg slope ($\phi - 2.5$ deg, as shown by the dashed line). Compared with horizontally placed PV modules, the one with optimum slope can generate 4.11% more power. For other orientations, one finding is that optimum tilt angles decrease into horizontal placement ($\beta = 0$ deg) accordingly. For BIPV systems, PV modules are usually placed according to the facade shapes and architecture arts, so that analyzing the yearly performance with azimuth and slope angles is quite valuable as illustrated in Fig. 2.

With the exception of west-facing and east-facing orientations, the yearly collected solar radiation decreases sharply when the slopes exceed 40 deg. If the PV modules have to be installed vertically to fit building facades, the yearly collected irradiance from the sun is 598.19 kWh/m^2 ($\gamma = -90$ deg), decreased by

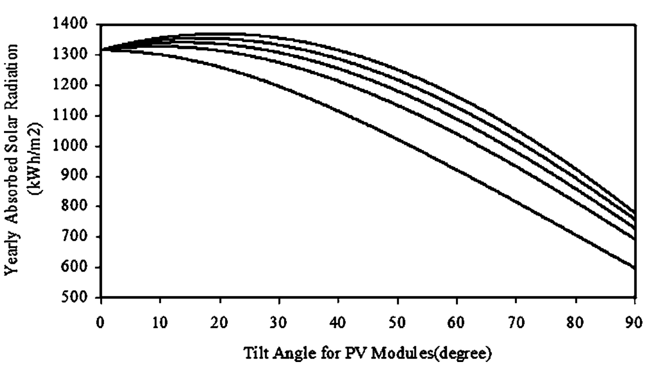

Fig. 2 Effect of tilt angles on yearly absorbed solar radiation on PV panels (upper to lower curves: $\gamma = 0$, −30, −45, −60, −90 deg)

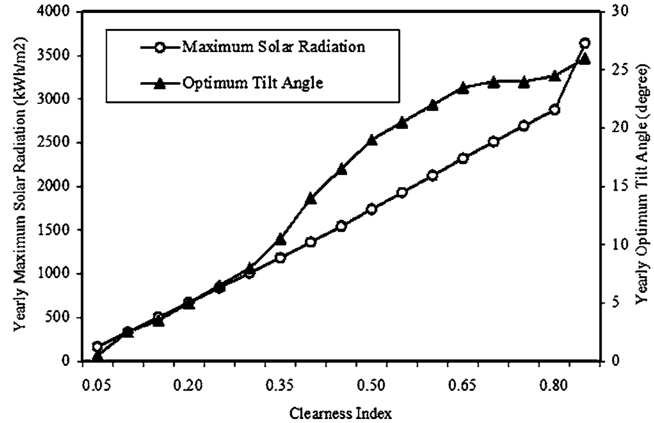

Fig. 3 Yearly optimum tilt angle and clearness index for south-facing PV facade

54.55% compared with the highest value of 1316.07 kWh/m^2. The vertical placement reduces nearly 50% power output of PV modules compared with the optimum option.

3.2 Seasonal and Monthly Optimum Tilt Angle. For stand-alone systems, system reliability is a very important factor. For most areas, winter is characterized by low irradiance. Thus, the most unreliable season can be set as the design benchmark. By considering the winter season (December, January, and February), the optimum slopes can be found. The maximum point occurs at south-facing orientation with the slope angle of 41 deg ($\phi + 18.5$ deg), while the value of yearly total irradiance drops by 4.32%.

If the slopes of PV modules can be adjusted monthly, or the panels are only utilized for certain months, the results for monthly optimum slopes are also different. The highest tilt angle appears with the value of 46 deg in December, while the angles become negative in May, June, and July.

3.3 Optimum Slopes Versus Condition Factors (k_T, G_0, ρ). Although the daily clearness index is random, the trend of monthly solar radiation density distributions is rather smooth. Both the G_0 and clearness index determine the sum of solar irradiance reaching the PV claddings. In spring, the average clearness index is lower, which leads to lower average monthly solar radiation. In April, the average clearness index is only 0.2423, and the highest occurs in October with the value of 0.4758. The yearly average clearness index is 0.3924.

If assuming the yearly clearness index to be constant, the yearly optimum tilt angle for south-facing panel increases as the increase of the value of clearness index, as shown in Fig. 3. The optimum angle is 14 deg ($\phi - 8.5$ deg) for yearly average clearness index of 0.4, 22 deg ($\phi - 0.5$ deg) for 0.6 clearness index, and 26 deg ($\phi + 3.5$ deg) for 1.0 clearness index. A polynomial function $\beta_{opt} = f(k_T)$ is found with high accuracy for the local ϕ. For the same latitude, β_{opt} is higher for clear sky weather.

4 Conclusions

One method is developed in this study to investigate the optimum orientations of BIPV claddings. The hourly clearness index is taken into account in the model. For BIPV systems in the Hong Kong area, their orientations affect their performance significantly, and the slopes exceeding 40 deg should be avoided. For stand-alone PV systems, the yearly optimum tilt angle for a south-facing azimuth is 20 deg, which is slightly less than the local latitude. Considering the worst solar season of winter, the optimum tilt angle reaches 41 deg (much higher than local latitude). If the slopes could be adjusted monthly, the energy performance would be much better.

Acknowledment

The work described in this paper was supported by the CLU Cross Departmental Project Fund of the Hong Kong Polytechnic University, China (Project No. 879R).

References

[1] Duffie J. A., and Bechman, W. A., 1980, *Solar Engineering of Thermal Processes*, John Wiley & Sons, New York.

[2] Lewis, G., 1987, "Optimum Tilt of Solar Collectors," Solar and Wind Energy, **4**, pp. 407–410.

[3] Asl-Soleimani, E., Farhangi, S., and Zabihi, M. S., 2001, "The Effect of Tilt Angle and Air Pollution on Performance of Photovoltaic Systems in Tehran," Renewable Energy, **24**, pp. 459–468.

[4] Christensen, C. B., and Barker, G. M., 2001, "Effects of Tilt and Azimuth on Annual Incident Solar Radiation for United States Locations, Solar Engineering 2001," S. J. Kleis and C. E. Bingham, eds., Proceedings of the International Solar Energy Conference Presented at FORUM 2001, 21–25 April 2001, Washington, DC, The American Society of Mechanical Engineers (ASME), New York, pp. 225–232; NREL Report No. CP-550-32966.

[5] Lu, L., 2004, "Investigation on Characteristics and Application of Hybrid Solar-Wind Power Generation Systems," Ph.D. thesis, The Hong Kong Polytechnic University, Kowlon, Hong Kong.

[6] Reindl, D. T., Beckman, W. A., and Duffie, J. A., 1990, "Evaluation of Hourly Titled Surface Radiation Models," Sol. Energy, **45**(1), pp. 9–17.

[7] Orgill, J. F., and Hollands, K. G. T., 1977, "Correlation Equation for Hourly Diffuse Radiation on a Horizontal Surface," Sol. Energy, **45**, pp. 357–359.

[8] Yik F. W. H., Chung, T. M., and Chan, K. T., 1995, "A Method to Estimate Direct and Diffuse Radiation in Hong Kong and its Accuracy," Hong Kong Inst. Eng. Trans., **2**(1), pp. 23–29.

[9] Yang, H. X., and Lu, L., 2004, "Study on Typical Meteorological Years and Their Effect on Building Energy and Renewable Energy Simulations," ASHRAE Trans., **110**(2), 424–431.